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SIMILARITY ANALYSIS FOR CHEMICAL
REACTORS AND THE SCALING OF LIQUID-
FUEL ROCKET ENGINES

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SIMILARITY ANALYSIS FOR CHEMICAL
REACTORS AND THE SCALING OF LIQUID-
FUEL ROCKET ENGINES*[†]

by

S. S. Penner

Daniel and Florence Guggenheim Jet Propulsion Center
California Institute of Technology
Pasadena, California

Scaling criteria are derived for homogeneous and heterogeneous chemical reactors from the conservation equations for multicomponent, reacting, gas mixtures, by the use of similarity analysis. A complete set of similarity parameters for homogeneous systems is shown to include Damköhler's first and third similarity constants, in addition to the parameters familiar from similarity studies on non-reacting gas mixtures. For low-velocity flow problems in the absence of external forces, the results of the present investigation are seen to be identical with Damköhler's five similarity criteria for chemical reactors.

Scaling of chemical reactors with complete similarity is seen to be practically impossible. Scaling with loss of geometric similarity, as well as scaling with loss of both geometric and dynamic similarity, is considered. Here a distinction must be drawn between similarity criteria for two extreme types of reactors, viz., (a) flow systems without heat transfer to the chamber walls and without wall-catalyzed chemical reactions;

* Supported by the Office of Ordnance Research, U. S. Army, under Contract DA 04-495-Ord-446. The author is indebted to Dr. H. S. Tsien for helpful comments.

† For a recent survey paper on Similarities in Combustion see Technical Report No. 15038-1 by A. E. Weller, R. E. Thomas, and B. A. Landry, Battelle Memorial Institute, June 1954. This paper does not contain detailed consideration of scaling problems for chemical reactors.

(b) flow systems with conductive heat transfer occurring only to the chamber walls and, for heterogeneous reactors, diffusive transport limited to lengths corresponding to the pore size of a catalyst bed. For case (a) obvious results are obtained for both homogeneous and heterogeneous reactors with loss of geometric similarity. Only case (b) has been treated before by Damköhler, Bosworth, and others; this work is reviewed here for reactors with loss of geometric similarity, as well as for reactors with loss of both geometric and dynamic similarity.

The results of this survey on similarity studies of chemical reactors are used to draw some qualitative conclusions regarding scaling criteria for liquid-fuel rocket engines. The practical importance of the analysis is seen to be limited insofar as reasonable conjectures about scaling procedures are possible only if the physico-chemical processes of the combustion reactions can be classified into rate-controlling chemical reaction steps for motors of different sizes. However, the analysis suggests a reasonable approach to the interpretation of experimental data in small-scale units. Once a sufficient number of experiments has been carried out to determine, for example, the relation between overall reaction rate and the physico-chemical design parameters (e.g., pressure, temperature, etc.), rational scaling to larger units should be possible.

I. INTRODUCTION

In a recent survey paper on combustion problems in liquid-fuel rocket engines¹ we emphasized the fact that one of the most important

¹ S. S. Penner and P. P. Datner, paper presented before the Fifth International Symposium on Combustion, Pittsburgh, Pa., September 1954 (in press).

practical problems facing the rocket development engineer is the scaling of tested models to engines of larger sizes. As a rule, the scaling methods employed do not permit adequate performance estimates of larger units. Consequently, engine scaling usually involves engine testing and development for every new unit. This procedure is clearly inefficient and wasteful in view of the tremendous expenditures involved in development work on large rocket motors.

Related scaling problems arise, of course, in chemical reactors other than liquid-fuel rocket engines. In a pioneering paper, G. Damköhler² described some years ago procedures for similarity analysis of both homogeneous and heterogeneous chemical reactors. It is the purpose of the present discussion to show a more satisfactory development of similarity criteria for flow problems with chemical reactions, to examine the relation between our results and Damköhler's celebrated five similarity criteria, and finally to illustrate the use of the similarity parameters for the determination of rational scaling procedures on chemical reactors.

In Section II we present a straightforward development of similarity criteria for gas reactions in flow systems. Starting with von Kármán's form of the conservation equations,³ we follow Tsien⁴ in introducing dimensionless variables. We then identify the dimensionless groups which multiply the reduced variables as the similarity parameters for the problem. For low-velocity flow problems without external forces, our results are shown to be equivalent to Damköhler's five similarity parameters.

² G. Damköhler, Z. Elektrochem. 42, 846 (1936).

³ Th. von Kármán, Sorbonne Lectures 1952-53, Paris, France.

⁴ H. S. Tsien, Princeton University Series on High-Speed Aerodynamics, Volume III, Section A (in press).

Scaling procedures for chemical reactors with negligible heat loss to the walls, and without surface-catalyzed reactions, are discussed in Section III. No useful results are obtained under the restriction of strict similarity; with loss of geometric similarity, obvious conditions for scaling are estimated.

In Section IV we follow Damköhler,¹ Bosworth,⁵ and others in using the physical meaning of Damköhler's similarity criteria for the deduction of intuitively appealing similarity parameters for scaling of reactors with heat loss to the motor walls and with surface-catalyzed chemical reactions. Scaling with loss of geometric similarity, as well as with loss of both geometric and flow similarity, is discussed.

The contents of Section V are restricted to some qualitative remarks concerning the problem of scaling for liquid-fuel rocket engines. No useful conclusions can be drawn without qualitative classification of reaction processes into homogeneous or heterogeneous reactions, and without some knowledge regarding the importance of heat transfer to the chamber walls or of wall-catalyzed chemical reactions. It is apparent that analytical studies of the type developed in the present discussion are useful only in connection with an intelligent experimental investigation of scaling problems in chemical reactors in general, and in engines in particular.

Before proceeding with the analysis, it is desirable to summarize briefly the physical contents of Damköhler's work. In this connection it is satisfying to note that Damköhler's five criteria are a necessary and sufficient condition for assuring dynamic and reaction-kinetic similarity

⁵ R. C. L. Bosworth, Trans. Faraday Soc. 43, 399 (1947).

in low-velocity flow problems without external forces and without heat loss to the motor walls.

From an examination of the conservation equations for a multi-component reacting gas mixture, Damköhler concluded that similarity of flow and reaction profile could be maintained only if the following five dimensionless ratios remained invariant:

$$(Re) = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$$(D_I) = \frac{\text{composition change produced by chemical reaction}}{\text{composition change produced by convection}}$$

$$(D_{II}) = \frac{\text{composition change produced by chemical reaction}}{\text{composition change produced by diffusion}}$$

$$(D_{III}) = \frac{\text{heat released by chemical reaction}}{\text{heat lost by convection}}$$

and

$$(D_{IV}) = \frac{\text{heat released by chemical reaction}}{\text{heat lost by conduction}}$$

II. SIMILARITY CRITERIA FOR REACTING GAS MIXTURES

A. Conservation Laws

All chemical processes are described completely by a set of conservation equations.* For chemical processes involving m distinct chemical species, the set of m continuity equations is represented by the relations³

$$\frac{\partial \gamma_i}{\partial t} = \frac{\omega_i}{\rho} - v_l \frac{\partial \gamma_i}{\partial x_l} - \frac{1}{\rho} \frac{\partial}{\partial x_l} (\rho \gamma_i V_{i,l}), \quad i=1,2,\dots,m, \quad (1)$$

* The conservation equations and similarity criteria are directly applicable to homogeneous gas mixtures and also to small volume elements of heterogeneous reactants, provided these volume elements are large compared to discontinuities in the heterogeneous reactants and provided we use suitable expressions for the transport properties in heterogeneous systems.

where a repeated index ℓ indicates summation over this index.[†] Here Y_i equals the weight fraction of species i , t is the time, ω_i represents the mass rate of production by chemical reaction of species i per unit volume, ρ is the density of the fluid mixture, \vec{V} equals the mass-weighted average velocity of the fluid mixture with components v_ℓ ($\ell = 1, 2, 3$), and \vec{V}_i is the diffusion velocity of species i with components $V_{i,\ell}$ ($\ell = 1, 2, 3$). An exact description of diffusion in multicomponent (gas) mixtures is a notably complex problem.⁶ For this reason it is necessary to simplify the analysis by treating the complex reaction mixture as a two-component system and writing³

$$V_{i,\ell} = - D_i \frac{\partial \ln Y_i}{\partial x_\ell} \quad (2)$$

where D_i is an appropriate binary diffusion coefficient for species i . It is unlikely that the simplification expressed by Eq. (2) can lead to erroneous answers in a similarity analysis. From Eqs. (1) and (2) it follows that

$$\frac{\partial Y_i}{\partial t} = \frac{\omega_i}{\rho} - v_\ell \frac{\partial Y_i}{\partial x_\ell} + \frac{1}{\rho} \frac{\partial}{\partial x_\ell} \left(\rho D_i \frac{\partial \ln Y_i}{\partial x_\ell} \right). \quad (3)$$

The momentum equation for a multicomponent gas mixture has been written by von Kármán³ in the form

$$\rho \left[\frac{\partial v_k}{\partial t} + (v_\ell \frac{\partial}{\partial x_\ell}) v_k \right] = - \frac{\partial p}{\partial x_k} + \rho F_k + \frac{\partial}{\partial x_\ell} (\tau_{\ell,k}^V + \tau_{\ell,k}^D), k=1,2,3. \quad (4)$$

[†] We use the indices $i, j = 1, 2, \dots, m$ for composition variables and $\ell, k, \ell' = 1, 2, 3$ for vector components.

⁶ J. O. Hirschfelder, C. F. Curtiss and R. B. Bird, The Molecular Theory of Gases and Liquids, John Wiley and Sons, Inc., New York 1954.

Here p represents the pressure; \vec{F} is an external force with components F_k ($k = 1, 2, 3$); τ^v and τ^D denote, respectively, the viscous (for Stokes flow) and diffusion stress tensors with components

$$\tau_{l,k}^v = \eta \left[-\frac{2}{3} \frac{\partial v_l}{\partial x_l} \delta_{l,k} + \left(\frac{\partial v_l}{\partial x_k} + \frac{\partial v_k}{\partial x_l} \right) \right] \quad (5)$$

and

$$\tau_{l,k}^D = -\rho \sum_{j=1}^m Y_j V_{j,l} V_{j,k} \approx -\rho \sum_{j=1}^m Y_j D_j^2 \frac{\partial \ln Y_j}{\partial x_l} \frac{\partial \ln Y_j}{\partial x_k} \quad (6)$$

The quantity η is taken to be the viscosity coefficient of the reacting mixture and $\delta_{l,k} = 1$ for $l = k$, $\delta_{l,k} = 0$ for $l \neq k$.

An approximate form of the energy equation, in which radiant heat transfer and thermal diffusion effects are neglected, is the following:³

$$\begin{aligned} \rho \frac{\partial}{\partial t} (e + \frac{1}{2} q^2) + \rho v_l \frac{\partial}{\partial x_l} (e + \frac{1}{2} q^2) + \rho \frac{\partial}{\partial t} \left(\frac{1}{2} \sum_{i=1}^m Y_i |V_i|^2 \right) \\ + \rho v_l \frac{\partial}{\partial x_l} \left(\frac{1}{2} \sum_{i=1}^m Y_i |V_i|^2 \right) = - \frac{\partial (p v_l)}{\partial x_l} + \frac{\partial}{\partial x_l} \left[v_k (\tau_{kl}^v + \tau_{kl}^D) \right] \\ + \frac{\partial}{\partial x_l} \left(\lambda \frac{\partial T}{\partial x_l} \right) + \rho F_l v_l - \frac{\partial}{\partial x_l} \left(\rho \sum_{i=1}^m Y_i h_i V_{i,l} \right) \\ - \frac{\partial}{\partial x_l} \left(\frac{1}{2} \rho \sum_{i=1}^m Y_i V_{i,l} |V_i|^2 \right). \end{aligned} \quad (7)$$

In Eq. (7) e represents the specific internal energy of the fluid mixture, $\frac{1}{2} q^2$ is the specific kinetic energy, λ is the thermal conductivity, h_i

is the specific enthalpy of the i 'th component, and the other symbols have their previous meaning. We define the standard specific energy e° and standard specific enthalpy h° by the relations

$$e = e^\circ + \bar{c}_v \vartheta$$

and

$$h = h^\circ + \bar{c}_p \vartheta$$

where

$$\bar{c}_v \vartheta = \int_{T^\circ}^T c_v dT, \quad \bar{c}_p = \gamma \bar{c}_v,$$

with \bar{c}_v and \bar{c}_p equal to the average specific heats of the reacting mixture at constant volume and constant pressure, respectively, for the indicated temperature range. The temperature ϑ equals, approximately, the temperature in excess of the reference temperature T° (usually 298.16°K) at which the standard internal energy and enthalpy have been evaluated. In view of the relation $e = e^\circ + \bar{c}_v \vartheta$ we note that

$$\begin{aligned} \rho \frac{\partial}{\partial t} (e + \frac{1}{2} q^2) + \rho v_\ell \frac{\partial}{\partial x_\ell} (e + \frac{1}{2} q^2) + \frac{\partial (p v_\ell)}{\partial x_\ell} \\ = \rho \frac{\partial}{\partial t} (\bar{c}_v \vartheta + \frac{1}{2} q^2) + \frac{\partial}{\partial x_\ell} \left[\rho (e + \frac{1}{2} q^2 + \frac{p}{\rho}) v_\ell \right] \\ - \rho (e + \frac{1}{2} q^2) \frac{\partial v_\ell}{\partial x_\ell}. \end{aligned}$$

From the overall continuity equation it follows that

$$\rho \frac{\partial v_\ell}{\partial x_\ell} = - \frac{\partial \rho}{\partial t}$$

whence

$$\begin{aligned} \rho \frac{\partial}{\partial t} (\bar{c}_v \vartheta + \frac{1}{2} q^2) - \rho e^0 \frac{\partial v_\ell}{\partial x_\ell} - \rho (\bar{c}_v \vartheta + \frac{1}{2} q^2) \frac{\partial v_\ell}{\partial x_\ell} \\ = \frac{\partial}{\partial t} [\rho (\bar{c}_v \vartheta + \frac{1}{2} q^2)] + e^0 \frac{\partial \rho}{\partial t} \end{aligned}$$

and

$$\begin{aligned} \rho \frac{\partial}{\partial t} (e + \frac{1}{2} q^2) + \rho v_\ell \frac{\partial}{\partial x_\ell} (e + \frac{1}{2} q^2) + \frac{\partial (p v_\ell)}{\partial x_\ell} \\ = \frac{\partial}{\partial t} [\rho (\bar{c}_v \vartheta + \frac{1}{2} q^2)] + e^0 \frac{\partial \rho}{\partial t} \\ + \frac{\partial}{\partial x_\ell} [\rho (h + \frac{1}{2} q^2) v_\ell]. \end{aligned}$$

Introduction of the preceding relation, as well as ^{of} Eq. (2), into Eq. (7)

leads to the result

$$\begin{aligned} \frac{\partial}{\partial t} [\rho (\bar{c}_v \vartheta + \frac{1}{2} q^2)] + e^0 \frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x_\ell} [\rho (h^0 + \frac{1}{2} q^2) v_\ell] - \frac{\partial}{\partial x_\ell} (\rho \bar{c}_p \cdot \tau_{\ell}) \\ + \frac{\partial}{\partial x_\ell} (\lambda \frac{\partial \vartheta}{\partial x_\ell}) + \frac{\partial}{\partial x_\ell} (v_k \tau_{k,\ell}^v) + \frac{\partial}{\partial x_\ell} (v_k \tau_{k,\ell}^D) \\ + \rho F_\ell v_\ell - \frac{\partial}{\partial x_\ell} (\rho \sum_{i=1}^m \gamma_i h_i D_i \frac{\partial \ln \gamma_i}{\partial x_\ell}) \\ - \rho \frac{\partial}{\partial t} [\frac{1}{2} \sum_{i=1}^m \gamma_i D_i^2 |\frac{\partial \ln \gamma_i}{\partial x_k}|^2] \\ - \rho v_\ell \frac{\partial}{\partial x_\ell} [\frac{1}{2} \sum_{i=1}^m \gamma_i D_i^2 |\frac{\partial \ln \gamma_i}{\partial x_k}|^2] \\ + \frac{\partial}{\partial x_\ell} [\frac{1}{2} \rho \sum_{i=1}^m \gamma_i D_i^3 \frac{\partial \ln \gamma_i}{\partial x_\ell} |\frac{\partial \ln \gamma_i}{\partial x_k}|^2]. \end{aligned} \quad (3)$$

Equation (8) is considerably more complex, and also more nearly correct, than Damköhler's form of the energy equation. Nevertheless, we expect to obtain essentially the same similarity criteria as Damköhler since the differences in the energy equation are the result of more refined descriptions of the same physical phenomena included in Damköhler's similarity studies.

B. Limitations of Similarity Analysis Based on the Conservation Laws

It was pointed out some years ago by Edgeworth-Johnstone⁷ that derivation of similarity criteria from the conservation equations will lead to more similarity parameters than are required, because the reaction frequencies $\omega_i/\rho Y_i$ are not independent but are functions, for example, of the composition, pressure and temperature distribution in the reactor. We are however, unable to take advantage of this fact in most practical scaling problems because the exact form of the reaction-rate law is usually unknown. In view of this difficulty, it is only reasonable to perform similarity studies on the assumption that useful conclusions can be drawn only if the results permit scaling for fixed values of $\omega_i/\rho Y_i$ under invariant composition, pressure, and temperature conditions. We shall interpret the utility of similarity studies for specific problems in Sections III and IV with this reservation in mind. However, in the discussion of engine scaling on the basis of model experiments (Section V), a different viewpoint is indicated, namely,

⁷ R. Edgeworth-Johnstone, Trans. Inst. Chem. Engrs. 17, 129 (1939).

fruitful experiments should be designed in such a way as to determine precisely this functional form in small units. Intelligent treatment of the data will then permit rational scaling by the use of similarity analysis.

C. Similarity Criteria

For the derivation of similarity criteria we follow Tsien's treatment for non-reacting gas mixtures⁴ and introduce dimensionless variables (identified by an asterisk) through the relations

$$\begin{aligned}
 Y_i &= Y_i^*, \quad t = t^* / U_i \quad \text{where} \quad U_i U_i^* = (\omega_i / \rho Y_i), \quad v_L = v_0 v_L^*, \\
 q &= v_0 q^*, \quad \rho = \rho_0 \rho^*, \quad x_L = L x_L^*, \quad P = P_0 P^*, \quad F_L = g F_L^*, \\
 \tau_{Lk}^V &= \frac{\eta_0 v_0}{L} \tau_{Lk}^{V,*}, \quad \tau_{Lk}^D = \frac{\rho_0 D_0^2}{L^2} \tau_{Lk}^{D,*}, \\
 h^0 &= (h_f^0 - h_0^0) h^{0,*} = q' h^{0,*}, \quad e^0 = h^0 - \frac{P}{\rho} = q' h^{0,*} - \frac{P_0}{\rho_0} \frac{P^*}{\rho^{*}}, \\
 h_i^0 &= q' h_i^{0,*}, \quad e_i^0 = q' e_i^{0,*} = q' h_i^{0,*} - \frac{P_0}{\rho_0} \frac{P^*}{\rho^{*}}, \\
 \lambda &= \lambda_0 \lambda^*, \quad \vartheta = \vartheta_0 \vartheta^*, \quad D_j = D_0 D_j^*, \\
 \bar{c}_v &= c_{p,0} \bar{c}_v^* / \gamma_0, \quad \bar{c}_p = c_{p,0} \bar{c}_p^*, \quad \text{where} \\
 \gamma_0 &= c_{p,0} / c_{v,0}.
 \end{aligned}
 \tag{9}$$

In Eq. (9) $U_{i,0} = (\omega_i / \rho Y_i)_0 \equiv U_i$ is a characteristic reaction frequency, the subscript 0 identifies properties of the initial gas mixture, L represents a characteristic length, g is the acceleration of gravity, $q' = h_f^0 - h_0^0$ is the standard specific enthalpy difference between reaction products and reactants and equals the heat evolved on reaction per gram at constant pressure and standard temperature, D_0 is a suitably chosen binary

diffusion coefficient of a dominant species in the inlet reaction mixture, and C_{p0} is the corresponding specific heat at constant pressure.

In terms of the dimensionless variables defined by Eq. (9),

Eq. (3) becomes

$$U_i \frac{\partial Y_i^*}{\partial t} = U_i U_i^* Y_i^* - v_{\ell}^* \frac{\partial Y_i^*}{L \partial x_{\ell}^*} + \frac{D_0}{L^2} \frac{1}{\rho^*} \frac{\partial}{\partial x_{\ell}^*} \left(\rho^* D_i^* \frac{\partial \ln Y_i^*}{\partial x_{\ell}^*} \right)$$

or

$$\frac{\partial Y_i^*}{\partial t} = U_i^* Y_i^* - \left(\frac{v_0}{L U_i} \right) v_{\ell}^* \frac{\partial Y_i^*}{\partial x_{\ell}^*} + \left(\frac{D_0}{U_i L^2} \right) \frac{1}{\rho^*} \frac{\partial}{\partial x_{\ell}^*} \left(\rho^* D_i^* \frac{\partial \ln Y_i^*}{\partial x_{\ell}^*} \right). \quad (10)$$

Equation (10) contains the dimensionless groups

$$D_1 = \left(\frac{U_i L}{v_0} \right) = \text{Damköhler's first dimensionless group}, \quad (11)$$

and

$$D_2 = \left(\frac{U_i L^2}{D_0} \right) = \text{Damköhler's second dimensionless group}. \quad (12)$$

Before proceeding with the derivation of other similarity criteria, it is desirable to elucidate the meaning of the reaction frequency U_i . Because of the independence of the chemical reactions in complex systems, we restrict analysis to the typical chemical species i . If the physical states of two systems are similar, and the time history of an important chemical species (namely, the species i) is also similar, then the entire reaction scheme in the two systems will usually be similar.

In terms of dimensionless variables Eq. (4) becomes

$$\rho_0 U_0 v_0 s^* \frac{\partial v_k^*}{\partial t^*} + \frac{\rho_0 v_0^2}{L} s^* \left(v_\ell^* \frac{\partial}{\partial x_\ell^*} \right) v_k^* = - \frac{p_0}{L} \frac{\partial p^*}{\partial x_\ell^*} \\ + \rho_0 g s^* F_k^* + \frac{\eta_0 v_0}{L^2} \frac{\partial}{\partial x_\ell^*} \tau_{\ell,k}^{v,*} \\ + \frac{\rho_0 D_0^2}{L^3} \frac{\partial}{\partial x_\ell^*} \tau_{\ell,k}^{D,*}$$

or

$$\left(\frac{U_0 L}{v_0} \right) s^* \frac{\partial v_k^*}{\partial t^*} + s^* \left(v_\ell^* \frac{\partial}{\partial x_\ell^*} \right) v_k^* = - \frac{1}{\gamma_0} \left(\frac{\rho_0 p_0}{\rho_0 v_0^2} \right) \frac{\partial p^*}{\partial x_\ell^*} \\ + \left(\frac{g L}{v_0^2} \right) F_k^* s^* + \left(\frac{\eta_0}{\rho_0 v_0 L} \right) \frac{\partial}{\partial x_\ell^*} \tau_{\ell,k}^{v,*} \\ + \left(\frac{\rho_0 D_0}{\eta_0} \right)^2 \left(\frac{\eta_0}{\rho_0 v_0 L} \right)^2 \frac{\partial}{\partial x_\ell^*} \tau_{\ell,k}^{D,*} \quad (13)$$

Equation (13) contains the following new dimensionless groups, in addition to the heat capacity ratio γ_0 :

$$M = \sqrt{\frac{\rho_0 v_0^2}{\gamma_0 p_0}} \quad = \text{Mach number,} \quad (14)$$

$$Fr = \left(\frac{v_0^2}{g L} \right) \quad = \text{Froude number,} \quad (15)$$

$$Re = \left(\frac{\rho_0 v_0 L}{\eta_0} \right) \quad = \text{Reynolds number,} \quad (16)$$

$$Sc = \left(\frac{\eta_0}{\rho_0 D_0} \right) \quad = \text{Schmidt number,} \quad (17)$$

It should be noted that the groups listed in Eqs. (11), (12), (16), and (17) are not independent and that any three of the ratios can be used to obtain the fourth.

In terms of dimensionless variables, Eq. (8) becomes

$$\begin{aligned}
 & \rho_0 (c_{p,0}/\rho_0) \partial_t U_i \frac{\partial}{\partial t^*} (\rho^* \bar{c}_v^* \vartheta^*) + \frac{1}{2} U_i \rho_0 v_0^2 \frac{\partial}{\partial t^*} (q^*)^2 + q' U_i \rho_0 h^{0,*} \frac{\partial \rho^*}{\partial t^*} \\
 & - U_i \rho_0 \frac{p^*}{\rho^*} \frac{\partial \rho^*}{\partial t^*} = - \frac{\rho_0 q' v_0}{L} \frac{\partial}{\partial x_{\ell}^*} (\rho^* h^{0,*} v_{\ell}^*) - \frac{1}{2} \frac{\rho_0 v_0^3}{L} \frac{\partial}{\partial x_{\ell}^*} (\rho^* q'^2 v_{\ell}^*) \\
 & - \frac{\rho_0 c_{p,0} \vartheta_0 v_0}{L} \frac{\partial}{\partial x_{\ell}^*} (\rho^* \bar{c}_p^* \vartheta^* v_{\ell}^*) + \frac{\lambda_0 \vartheta_0}{L^2} \frac{\partial}{\partial x_{\ell}^*} (\lambda^* \frac{\partial \vartheta^*}{\partial x_{\ell}^*}) \\
 & + \frac{\eta_0 v_0^2}{L^2} \frac{\partial}{\partial x_{\ell}^*} (v_k^* \gamma_{k,\ell} v_{\ell}^*) + \frac{\rho_0 v_0 D_0^2}{L^3} \frac{\partial}{\partial x_{\ell}^*} (v_k^* \gamma_{k,\ell} D_{\ell}^*) \\
 & + \rho_0 g v_0 \rho^* F_{\ell}^* v_{\ell}^* - \frac{\rho_0 q' D_0}{L^2} \frac{\partial}{\partial x_{\ell}^*} (\rho^* \sum_{i=1}^m \gamma_i^* h_i^{0,*} D_i^* \frac{\partial \ln \gamma_i^*}{\partial x_{\ell}^*}) \\
 & - \frac{U_i \rho_0 D_0^2}{L^2} \rho^* \frac{\partial}{\partial t^*} \left[\frac{1}{2} \sum_{i=1}^m \gamma_i^* D_i^{*2} \left| \frac{\partial \ln \gamma_i^*}{\partial x_{\ell}^*} \right|^2 \right] \\
 & - \frac{\rho_0 v_0 D_0^2}{L^3} \rho^* v_{\ell}^* \frac{\partial}{\partial x_{\ell}^*} \left[\frac{1}{2} \sum_{i=1}^m \gamma_i^* D_i^{*2} \left| \frac{\partial \ln \gamma_i^*}{\partial x_{\ell}^*} \right|^2 \right] \\
 & - \frac{\rho_0 D_0^3}{L^4} \frac{\partial}{\partial x_{\ell}^*} \left[\frac{1}{2} \rho^* \sum_{i=1}^m \gamma_i^* D_i^{*3} \left(\frac{\partial \ln \gamma_i^*}{\partial x_{\ell}^*} \right) \left| \frac{\partial \ln \gamma_i^*}{\partial x_{\ell}^*} \right|^2 \right]
 \end{aligned}$$

or

$$\begin{aligned}
& \frac{1}{\gamma_0} \frac{\partial}{\partial t^*} (S^* \bar{c}_v^* \vartheta^*) + \left(\frac{\frac{1}{2} v_0^2}{c_{p,0} \vartheta_0} \right) \frac{\partial}{\partial t^*} (q^*)^2 + \left(\frac{v_0}{U_i L} \right) \left(\frac{q' U_i L}{v_0 c_{p,0} \vartheta_0} \right) h^{0,*} \frac{\partial S^*}{\partial t^*} \\
& - \left(\frac{P_0}{\rho_0 v_0^2} \right) \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \frac{P^*}{S^*} \frac{\partial S^*}{\partial t^*} = - \left(\frac{v_0}{L U_i} \right)^2 \left(\frac{q' U_i L}{v_0 c_{p,0} \vartheta_0} \right) \frac{\partial}{\partial x_\ell^*} (S^* h^{0,*} v_\ell^*) \\
& - \frac{1}{2} \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{v_0}{L U_i} \right) \frac{\partial}{\partial x_\ell^*} (S^* q^{*2} v_\ell^*) \\
& - \left(\frac{v_0}{L U_i} \right) \frac{\partial}{\partial x_\ell^*} (S^* \bar{c}_p^* \vartheta^* v_\ell^*) \\
& + \left(\frac{\eta_0}{\rho_0 D_0} \right) \left(\frac{\lambda_0}{c_{p,0} \eta_0} \right) \left(\frac{D_0}{U_i L^2} \right) \frac{\partial}{\partial x_\ell^*} (\lambda^* \frac{\partial \vartheta^*}{\partial x_\ell^*}) \\
& + \left(\frac{\eta_0}{\rho_0 v_0 L} \right) \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{v_0}{L U_i} \right) \frac{\partial}{\partial x_\ell^*} (v_k^* \gamma_{k,e}^*) \\
& + \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{\rho_0 D_0}{\eta_0} \right)^2 \left(\frac{\eta_0}{\rho_0 v_0 L} \right)^2 \left(\frac{v_0}{U_i L} \right) \frac{\partial}{\partial x_\ell^*} (v_k^* \gamma_{k,i}^*) \\
& + \left(\frac{q' L}{v_0^2} \right) \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{v_0}{L U_i} \right) S^* F_\ell^* v_\ell^* \\
& - \left(\frac{q' U_i L}{v_0 c_{p,0} \vartheta_0} \right) \left(\frac{v_0}{L U_i} \right)^2 \left(\frac{\rho_0 D_0}{\eta_0} \right) \left(\frac{\eta_0}{\rho_0 v_0 L} \right) \frac{\partial}{\partial x_\ell^*} \left(S^* \sum_{i=1}^m \gamma_i^* h_{i,e}^{0,*} \frac{\partial \ln \gamma_i^*}{\partial x_\ell^*} \right) \\
& - \left(\frac{\rho_0 D_0}{\eta_0} \right)^2 \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{\eta_0}{\rho_0 v_0 L} \right)^2 S^* \frac{\partial}{\partial t^*} \left[\frac{1}{2} \sum_{i=1}^m \gamma_i^* D_i^{*2} \left| \frac{\partial \ln \gamma_i^*}{\partial x_k^*} \right|^2 \right] \\
& - \left(\frac{D_0}{L^2 U_i} \right) \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \left(\frac{D_0 \rho_0}{\eta_0} \right) \left(\frac{\eta_0}{\rho_0 v_0 L} \right) S^* v_\ell^* \frac{\partial}{\partial x_\ell^*} \left[\frac{1}{2} \sum_{i=1}^m \gamma_i^* D_i^* \left| \frac{\partial \ln \gamma_i^*}{\partial x_k^*} \right|^2 \right] \\
& + \left(\frac{\rho_0 D_0}{\eta_0} \right) \left(\frac{\eta_0}{\rho_0 v_0 L} \right) \left(\frac{L U_i}{v_0} \right) \left(\frac{D_0}{L^2 U_i} \right)^2 \left(\frac{v_0^2}{c_{p,0} \vartheta_0} \right) \frac{\partial}{\partial x_\ell^*} \left[\frac{1}{2} S^* \sum_{i=1}^m \gamma_i^* \gamma_i^{*2} \right. \\
& \quad \left. \times \left(\frac{\partial \ln \gamma_i^*}{\partial x_\ell^*} \right) \left| \frac{\partial \ln \gamma_i^*}{\partial x_k^*} \right|^2 \right]. \quad (13)
\end{aligned}$$

Reference to Eq. (18) shows that the following new dimensionless groups appear:

$$D_{III} = \left(\frac{q' U_i L}{v_o c_{p,o} \mathcal{D}_o} \right) = \text{Damköhler's third dimensionless group,} \quad (19)$$

$$\frac{v_o^2}{c_{p,o} \mathcal{D}_o} > \gamma_o, \quad (20)$$

$$Pr = \left(\frac{c_{p,o} \eta_o}{\lambda_o} \right) = \text{Prandtl number,} \quad (21)$$

For convenience and later reference we summarize an independent set of similarity parameters for reacting multicomponent gas mixtures in which radiant heat transfer, thermal diffusion, and other relatively unimportant transport processes are ignored. The important relations are the following:

$$Re = \frac{\rho_o v_o L}{\eta_o}, \quad (I)$$

$$Sc = \frac{\eta_o}{\rho_o \mathcal{D}_o}, \quad (II)$$

$$Pr = \frac{c_{p,o} \eta_o}{\lambda_o}, \quad (III)$$

$$M = \sqrt{\frac{\rho_o v_o^2}{\gamma_o p_o}}, \quad (IV)$$

$$Fr = \frac{v_o^2}{g L}, \quad (V)$$

$$D_I = \frac{L U_i}{v_o}, \quad (\text{VI})$$

$$D_{III} = \frac{q' U_i L}{v_o c_{p,o} \mathcal{D}_o}, \quad (\text{VII})$$

$$\varphi = \frac{\frac{1}{2} v_o^2}{(c_{p,o}/\gamma_o) \mathcal{D}_o}, \quad (\text{VIII})$$

$$\gamma_o. \quad (\text{IX})$$

The groups (I) to (V), (VIII), and (IX) must be maintained constant even in flow systems without chemical reaction.⁴ For low velocity flows, maintenance of constant values of M and φ is unimportant; in the absence of significant external forces, (i.e., forces of the type which might occur in combustion chambers during accelerated flight) constancy of Fr is unimportant; the specific heat ratio γ_o generally does not vary greatly from one chemical system to another and is, therefore, practically constant in any case. Hence it follows that constancy of the five groups (Re) , (Sc) , (Pr) , (D_I) , and (D_{III}) is generally sufficient to assure similar combustion processes in low-velocity systems without significant external forces since, in this case, the two different combustion reactions are described by identical nondimensional differential equations. The five similarity requirements given here are equivalent to Damköhler's set of (Re) , (D_I) , $(D_{II}) = (D_I)(Sc)(Re)^{1/2}$, (D_{III}) , and $(D_{IV}) = (D_{III})(Pr)^{1/2}(Re) = (D_{III})(Pe)$ where the Peclet number (Pe) is known to play an im-

portant role in heat conduction with forced convection. We now proceed to investigate the rational limitations placed on the scaling of chemical reactors through the requirement that (Re) , (Sc) , (Pr) , (D_I) , and (D_{III}) must be kept constant, after considering briefly the influence of boundary conditions.

D. Boundary Conditions

For reactors in which heat transfer to the motor wall plays an important role we must introduce a new similarity criterion which refers to the solid-gas interface. This effect is well known and it is shown, for example, by Teien,⁴ that the Nusselt heat transfer number is a suitable parameter. Actually we expect constant Nusselt number in reactors for fixed values of the Reynolds and Prandtl numbers. It is therefore apparent that the boundary condition corresponding to heat transfer to the motor wall introduces no new similarity parameter in the present problem.

Wall-catalyzed chemical reactions introduce a new reaction frequency into the similarity study. We shall assume throughout the following discussion that the reaction chambers are sufficiently large to justify neglect of wall-catalyzed chemical reactions.

III. SCALING PROCEDURES FOR CHEMICAL REACTORS WITHOUT DOMINANT HEAT LOSS TO THE CHAMBER WALLS AND WITHOUT WALL-CATALYZED REACTIONS*

For low-velocity flow problems in the absence of external forces, and without dominant heat transfer to the chamber walls, we use the similarity parameters (Re) , (Sc) , (Pr) , (D_I) , and (D_{III}) directly. The following discussion will be restricted to scaling with exact similarity and to scaling with the loss of geometric similarity.

Consider a cylindrical model M and a large-scale reactor H . The consumption ratio (or total volume flow rate ratio) n for H , with respect to M as reference reactor, is then defined by the relation

$$n = d_H^2 v_{0,H} / d_M^2 v_{0,M} \quad (22)$$

where d_H and d_M denote diameters in the reactors H and M , respectively. It is convenient to investigate the implications of the similarity requirements for fixed values of the consumption ratio n .

A. Maintenance of Exact Similarity for a Given Chemical System

Maintenance of exact similarity is impractical for both homogeneous and heterogeneous systems.

(1) Homogeneous Reactors

For a given chemical reaction, the physico-chemical parameters in similar reactors remain unchanged, i.e., g_0 , $c_{p,0}$, D_0 , η_0 , q' .

* For a discussion of the phrase "dominant heat loss" see Section IV.

and λ_0 are fixed. If the consumption rate is increased by the factor n , then it follows from Eq. (22) that

$$d_H^2 v_{0,H} = n d_M^2 v_{0,M} \quad (23)$$

From the requirement $(Re)_H = (Re)_M$ it is apparent that*

$$d_H v_{0,H} = d_M v_{0,M} \quad (24)$$

where the characteristic length occurring in the Reynold's number has been set equal to the diameter for a homogeneous cylindrical reactor. From Eqs. (23) and (24) follow the relations

$$d_H = n d_M \quad (25)$$

and

$$v_{0,H} = \frac{1}{n} v_{0,M} \quad (26)$$

If l is a chamber length, then geometric similarity leads to the relation

$$l_H = n l_M \quad (27)$$

in view of Eq. (25). The requirements $(Sc)_H = (Sc)_M$ and $(Pr)_H = (Pr)_M$ lead to no restriction in the scaling of chemical reactors for a specified chemical reaction. The relation $(D_I)_H = (D_I)_M$ leads to the conclusion

$$(U_i)_H = \frac{1}{n^2} (U_i)_M \quad (28)$$

whereas $(D_{III})_H = (D_{III})_M$ shows now that

$$(g_c)_H = (g_c)_M \quad (29)$$

For $n > 1$ it is apparent from Eqs. (25) and (27) that reactor

* For scaling with maintenance of geometric similarity it is unimportant to differentiate between the different lengths. We specify the particular linear dimension involved only as a matter of convenience.

diameter and length must be increased; Eq. (26) shows that the flow velocity must be decreased; Eqs. (28) and (29) show that for fixed $(\vartheta_0)_H = (\vartheta_0)_M$ the reaction frequency must be reduced. The requirements for exact similarity expressed by Eqs. (25) to (29) are difficult or impossible to satisfy since we have no effective control over homogeneous chemical reaction rates. Furthermore, the indicated scaling procedure is very inefficient; we reduce the flow velocity and then compensate for this, in order to increase the volume flow rate by the factor n , by building a chamber of larger diameter; the chamber length is changed in such a way that the residence time l_H/v_H has been increased by the factor n^2 in order to allow for the reduction in reaction frequency by the factor $1/n^2$. These changes will lead to the same combustion efficiency in the two reactors.

(2) Heterogeneous Reactors

From Eq. (22) it follows that

$$d_H^2 v_{0,H} = n d_M^2 v_{0,M}$$

whereas the requirement $(Re)_H = (Re)_M$ leads to the condition

$$d_H' v_{0,H} = d_M' v_{0,M} \quad (30)$$

where d' represents the spacing between the discontinuities in the heterogeneous system. For exact similarity

$$d_H/d_M = d_H'/d_M' \quad (31)$$

whence

$$\sqrt{n v_{0,M}/v_{0,H}} = v_{0,M}/v_{0,H}$$

and

$$v_{0,H} = v_{0,M}/n \quad (32)$$

as before. Furthermore,

$$d_H = n d_M, d_H' = n d_M'. \quad (33)$$

Again, from geometric similarity,

$$l_H = n l_M. \quad (34)$$

Proceeding as for homogeneous systems we find

$$(U_i)_H = \frac{1}{n^2} (U_i)_M \quad (35)$$

and

$$(g_o)_H = (g_o)_M. \quad (36)$$

Thus exact similarity can be maintained for a heterogeneous reactor only if the reaction frequency is reduced by the factor n^2 when the linear dimensions (including the pore size of a catalyst bed) are increased by the factor n . This particular dependence of reaction frequency on pore diameter is usually impossible to obtain.

B. Geometric Distortion

For some homogeneous chemical reactions large-scale diffusion is not important. For heterogeneous chemical reactions in catalyst beds, and also for heterogeneous diffusion flames, the type of diffusion occurring between macroscopic volume elements can be neglected. Hence in these cases the diffusion terms in the continuity and energy equations disappear and the similarity requirement for (Sc) may be deleted.

For reactors with possible geometric distortion, somewhat different results are obtained for homogeneous and for heterogeneous systems. If geometric distortions are introduced, care must be taken to use for L an appropriate diameter, pore size or length. Thus we note that in the Reynold's number Re , L = chamber diameter d for a homogeneous

reactor, L = pore size d' in a catalyst bed, and L = spacing between heterogeneous diffusion flames d' in a liquid-gas system; in the group D_I the length L is a chamber length ℓ since the term L/v_0 represents a residence time; in the group D_{III} the length L represents also a chamber length since the method of introduction of this group shows that it must be commensurate with the length entering into D_I .

(1) Homogeneous Reactors

If the consumption rate for a given chemical system is increased by the factor n we have again the relation

$$d_H^2 v_{0,H} = n d_M^2 v_{0,M}.$$

From

$$(Re)_H = (Re)_M$$

it follows, as before, that

$$d_H v_{0,H} = d_M v_{0,M}$$

whence

$$d_H = n d_M \quad (37)$$

and

$$v_{0,H} = \frac{1}{n} v_{0,M}. \quad (38)$$

For fixed chemical systems the conditions $(Sc) = \text{constant}$ and $(Pr) = \text{constant}$ are satisfied automatically. From $(D_I)_H = (D_I)_M$ it follows now that

$$(U_\lambda \ell)_H = \frac{1}{n} (U_\lambda \ell)_M \quad (39)$$

whereas $(D_{II})_H = (D_{II})_M$ leads again to the result

$$(\mathcal{D}_0)_H = (\mathcal{D}_0)_M. \quad (40)$$

Without geometric distortion, Eq. (39) leads to the same results as were obtained in Section IIIA. However, other solutions are possible, the most interesting being

$$(U_i)_H = (U_i)_M \quad (41)$$

with

$$l_H = \frac{1}{n} l_M. \quad (42)$$

The prescription for scaling of homogeneous reactors, without geometric similarity or wall-catalyzed reactions, is hardly surprising. We note that residence time (l/v_0), reaction frequency, and temperature distribution are maintained uniform, with the chamber diameter and length scaled in inverse proportions.

(2) Heterogeneous Reactors With Constant Pore Size

We start again with the condition

$$d_H^2 v_{0,H} = n d_M^2 v_{0,M}. \quad (43)$$

From

$$(Re)_H = (Re)_M$$

or

$$\frac{\rho_0 d_H' v_{0,H}}{\eta_0} = \frac{\rho_0 d_M' v_{0,M}}{\eta_0}$$

we now find

$$v_{0,H} = v_{0,M} \quad (44)$$

if $d_H' = d_M'$ represents a constant pore size for a catalyst reactor, or a constant spacing between heterogeneous diffusion flames in a liquid-gas system. Equations (43) and (44) now lead to the result

$$d_H = \sqrt{n} d_M. \quad (45)$$

From the conditions $(Sc)_H = (Sc)_M$ and $(Pr)_H = (Pr)_M$ we again obtain no design restrictions for a given chemical system. In the present case

$$(D_I)_H = (D_I)_M \quad \text{and} \quad (D_{III})_H = (D_{III})_M$$

imply again the conditions

$$(U_i \ell)_H = (U_i \ell)_M \quad (46)$$

and

$$(\mathcal{D}_0)_H = (\mathcal{D}_0)_M. \quad (47)$$

For a given chemical system the solutions

$$(U_i)_H = (U_i)_M \quad (48)$$

and

$$\ell_H = \ell_M \quad (49)$$

are thus seen to be possible. The conclusion that a heterogeneous reactor with negligible heat loss to the walls, and without wall-catalyzed reactions, can be scaled with maintenance of flow, thermal, and reaction-kinetic similarity by increasing d with d' , ρ , U_i , and initial conditions constant, is hardly surprising. The assumptions introduced into the analysis imply no interference between adjacent macroscopic volumes of reactor; accordingly, the scaling procedure suggests design of large reactors by arranging small reactors in parallel. This scaling principle for heterogeneous reactors is known to lead frequently to very undesirable results because heat losses to the chamber walls may become very important. For this reason we return now to Damköhler's work and modify the similarity criteria on the assumption that heat losses to the chamber walls play a dominant role.

IV. SCALING CRITERIA FOR HOMOGENEOUS AND HETEROGENEOUS REACTORS WITH DOMINANT HEAT LOSS TO THE CHAMBER WALLS

Damköhler² and Bosworth⁵ start their investigations of scaling criteria with the physical contents of Damköhler's five similarity criteria and assume, for both homogeneous and heterogeneous reactors, that heat losses to the chamber walls are dominant. We shall review this work with special reference to Damköhler's investigations of partial similarity.

A. Homogeneous Reactors

For homogeneous reactors the following scaling criteria are assumed to apply:

$$Re = \frac{\rho_0 v_0 d}{\eta_0}, \quad (50)$$

$$D_I = \frac{U_i l}{v_0}, \quad (51)$$

$$D_{II} = \frac{U_i l d}{D_0}, \quad (52)$$

$$D_{III} = \frac{q' U_i l}{v_0 c_{p,0} \theta_0}, \quad (53)$$

and

$$D_{IV} = \frac{\rho_0 q' U_i d^2}{\lambda \theta_0}. \quad (54)$$

The groups (Re) to (D_{II}) correspond directly to the ratios given in the introduction, where they were expressed in terms of the physical contents of the dimensionless ratios. They were obtained by Damköhler from the conservation laws by introducing dimensionless variables and determining the groups which multiply the dimensionless ratios of inertial force to viscous force, of composition change produced by chemical reaction to composition change produced by convection, etc. The product ld arises in Eq. (52) from the notion that both radial and axial diffusion are important. The product of lengths d^2 occurs in Eq. (54) because we assume that all conductive heat losses occur to the chamber wall and normal to the direction of flow.

If heat transfer to the chamber wall occurs by turbulent convection, then $\bar{\lambda}$ in Eq. (54) should be replaced by the product of a Nusselt number and the gas conductivity, where the Nusselt number depends on the product of appropriate powers of (Pr) and (Re) .

(1) Homogeneous Reactor with Conductive Heat Transfer to the Chamber Walls (Complete Similarity)

From the consumption relation

$$d_H^2 v_{0,H} = n d_M^2 v_{0,M} \quad (55)$$

and the condition $(Re)_H = (Re)_M$ or

$$d_H v_{0,H} = d_M v_{0,M} \quad (56)$$

it follows that

$$d_H = n d_M \quad (57)$$

and

$$v_{0,H} = \frac{1}{n} v_{0,M}. \quad (58)$$

Hence it follows from $(D_I)_H = (D_I)_M$ or from $(D_{II})_H = (D_{II})_M$ that

$$(U_i)_H l_H = (U_i)_M l_M / n. \quad (59)$$

The requirement $(D_{III})_H / (D_{IV})_H = (D_{III})_M / (D_{IV})_M$ implies the condition

$$l_H = n l_M \quad (60)$$

for constant values of $\bar{\lambda}$.^{*} From Eqs. (59) and (60) it follows now that

$$(U_i)_H = \frac{1}{n^2} (U_i)_M. \quad (61)$$

Constancy of (D_{III}) or (D_{IV}) is seen to imply only constancy of \mathcal{D}_0 .

The scaling requirements embodied in Eqs. (57), (60), and (61) are practically impossible to fulfill for homogeneous reactors; they are equivalent to those obtained for maintenance of complete similarity without dominant heat losses to the chamber wall [compare Section III A (1)].

(2) Homogeneous Reactor with Heat Transfer by Turbulent Convection to the Chamber Walls (Complete Similarity)

Equations (50) to (53) remain unchanged; Eq. (54) is replaced by the expression

$$D_{IV} = \frac{\rho_0 q' U_i d^2}{\bar{\lambda} \mathcal{D}_0 (Nu)} \quad (62)$$

where Nu is the Nusselt number. For fixed values of the physico-chemical parameters of the gas, the Prandtl number is constant and the Nusselt number is a function only of the Reynolds number, i.e.,

$$Nu = \text{constant} (\rho_0 v_0 d / \eta_0)^{m'} \quad (63)$$

where $m' \approx 0.8$ for turbulent flow.

* Note that the relation $l_H = n l_M$ does not follow from assumed geometric similarity but is required by the similarity criteria directly. We cannot fulfill constancy of (Re) to (D_{IV}) without geometric similarity.

Proceeding as for homogeneous reactors, we find again

$$d_H = n d_M, \quad (64)$$

$$v_{0,H} = \frac{1}{n} v_{0,M}, \quad (65)$$

and

$$(U_i)_H l_H = (U_i)_M l_M / n. \quad (66)$$

Equation (66) and the condition $(D_{III})_H = (D_{III})_M$ show that

$$(D_0)_H = (D_0)_M. \quad (67)$$

According to Eqs. (62) and (63), constancy of (D_{IV}) again leads to the conclusion that

$$(U_i)_H = \frac{1}{n^2} (U_i)_M \quad (68)$$

since $(\rho_0 d v_0 / \eta_0)_H = (\rho_0 d v_0 / \eta_0)_M$. Hence also

$$l_H = n l_M \quad (69)$$

and we have again scaling criteria which are almost impossible to satisfy in practice.

B. Heterogeneous Reactors

Appropriate similarity criteria are

$$Re = \frac{\rho_0 v_0 d'}{\eta_0}, \quad (70)$$

$$D_I = \frac{l U_i}{v_0}, \quad (71)$$

$$D_{II} = \frac{U_i (d')^2}{D_0} \quad (72)$$

$$D_{III} = \frac{q' U_i l}{v_0 c_{p,0} \vartheta_0}, \quad (73)$$

and

$$D_{IV} = \frac{\beta_0 q' U_i d^2}{\bar{\lambda} \vartheta_0}. \quad (74)$$

The length d' in Eq. (70) arises from the notion that diffusion transport is important only within distances corresponding to the pore size of a catalyst reactor; the chamber diameter d occurs in Eq. (74) because we have introduced the assumption that only heat loss by conduction normal to the chamber axis is of importance. It is obvious, without detailed study, that Eqs. (70) to (74) will lead to results which are identical with those discussed in Section III B (2) for exact geometric similarity.

In his study of catalyst reactors, Bosworth⁵ has modified Eq. (74) further by writing

$$\bar{\lambda} = \lambda_0 (1 + \beta d') \quad (75)$$

where λ_0 represents the thermal conductivity of the gas within the catalyst pores at the temperature ϑ_0 and β is a proportionality constant which measures the contribution of radiant heat transfer (for low radiant intensities the pathlength for radiation is directly proportional to d'). The implications of Eqs. (70) to (75) for values of $\beta d'$ either small or large compared to unity have been discussed by Bosworth⁵ and will not be reproduced here.

The thermal conductivity $\bar{\lambda}$ appearing in Eq. (74) is an effective thermal conductivity for the catalyst reactor and can be expressed, for fixed Prandtl number, according to experimental studies carried out by Colburn,⁸ in the form

⁸ A. P. Colburn, Ind. Eng. Chem. 23, 910 (1931); Th. Chilton and A. P. Colburn, *ibid.*, 23, 913 (1931).

$$\bar{\lambda} = (Re/1000)^{m'} f(d'/d), 0.75 \leq m' \leq 0.97 \quad (76)$$

for turbulent flow. The function $f(d'/d)$ decreases from about 1.45 at $d'/d = 0.04$ to 0.2 at $d'/d = 0.3$. For exact similarity, both Re and d'/d are constants and, therefore, $\bar{\lambda}$ is constant. For $d'/d \ll 1$, $f(d'/d) \approx \text{constant}$.

It will now be shown that Eqs. (70) to (74) and (76) do not lead to useful scaling procedures even with the loss of geometric similarity.

(1) Loss of Geometric Similarity for a Catalyst Reactor with Constant Pore Size, Ignoring Mass Transport by Diffusion

From the definition for the consumption ratio we have the relation

$$d_H^2 v_{o,H} = n d_M^2 v_{o,M}.$$

Constancy of the Reynolds number implies the result

$$v_{o,H} = v_{o,M} \quad (77)$$

for constant pore size $d'_H = d'_M$. Therefore it follows that

$$d_H = \sqrt{n} d_M. \quad (78)$$

From the condition $(D_I)_H = (D_I)_M$ we now obtain the result

$$(U_i)_H l_H = (U_i)_M l_M; \quad (79)$$

we ignore the condition $(D_{II})_H = (D_{II})_M$; $(D_{III})_H = (D_{III})_M$ shows that

$$(v_o)_H = (v_o)_M \quad (80)$$

in view of Eq. (79). Finally, the requirement $(D_{IV})_H = (D_{IV})_M$ with constant value of $\bar{\lambda}$ (Re constant, $d'/d \ll 1$) shows that

$$(U_i)_H (d_H)^2 = (U_i)_M (d_M)^2. \quad (81)$$

From Eqs. (78) and (81) it follows now that

$$(U_i)_H = \frac{1}{n} (U_i)_M \quad (82)$$

whereas Eqs. (79) and (82) lead to the conclusion that

$$l_H = n l_M. \quad (83)$$

The similarity requirement expressed by Eq. (82) for a catalyst reactor with constant pore size is generally impossible to fulfill, i.e., the scaling of catalyst reactors for the design restrictions expressed by Eqs. (70) to (72), (74), and (76) cannot be carried out with maintenance of dynamic, thermal, and reaction-kinetic similarity. For this reason, we turn our attention now to Damköhler's methods of scaling with partial similarity.

C. Partial Similarity with Heat Loss to the Chamber Walls

Damköhler² has considered cases of partial similarity by specifying conditions for similar heat losses to the chamber wall, as well as similar heat release and reaction-rate patterns. Design criteria to yield similar heat loss conditions will now be considered. Following Damköhler,² we shall mean by thermal similarity conditions under which a fixed fraction of the reaction heat is lost to the wall.

If the mean reaction frequency in the chamber is \bar{U} , then the rate of heat release is

$$\bar{q}' \bar{U} \pi l d^2/4.$$

If $\vartheta_f - \vartheta_0$ represents the temperature change in the reaction vessel, then the convective heat loss per unit time from the flowing medium is, approximately,

$$\bar{c}_p \bar{q}' (\vartheta_f - \vartheta_0) \bar{v} \pi d^2/4.$$

Hence the ratio of total heat release to heat loss by convection is

$$\frac{q' \bar{U} l}{\bar{v} \bar{c}_p (\vartheta_f - \vartheta_0)} = D_{III}' \quad (84)$$

Constancy of (D_{III}') now replaces constancy of (D_{III}).

The heat release per unit length of combustion chamber is

$$\bar{g} q' \bar{U} \pi d^2 / 4.$$

Let ψ denote the fractional heat loss to the chamber by conduction only.

For fixed values of ψ it is then easily shown from the differential equation for heat conduction

$$\lambda \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \vartheta}{\partial r} \right) = - \bar{g} q' \bar{U} \psi$$

that

$$\frac{\bar{g} q' \bar{U} d^2}{\lambda (\vartheta_m - \vartheta_w)} = D_{IV}' \quad (85)$$

where ϑ_m and ϑ_w denote, respectively, the temperatures in the center of the reactor and at the wall. The similarity condition expressed by Eq.

(85) replaces the requirements for constant Peclet number. The group (D_{IV}') is obtained from (D_{III}') by multiplication with

$$(d/l)(Pr)^{1/2}(Re)[(\vartheta_f - \vartheta_o)/(\vartheta_m - \vartheta_w)].$$

Reaction-kinetic similarity is maintained by requiring constancy of (D_I).

Flow similarity is not maintained since constant (Re) and (Sc) are not imposed.

For turbulent flow, heat transport to the wall is characterized by a heat transmission coefficient α . If again a constant fraction of the heat released is lost to the wall then

$$\alpha \pi d (\vartheta_m - \vartheta_w) \sim \psi \bar{g} q' \bar{U} \pi d^2 / 4$$

and

$$\frac{4}{\psi} \sim \frac{\bar{g} q' \bar{U} d^2}{\lambda (\vartheta_m - \vartheta_w) (\alpha d / \lambda)} = \frac{D_{IV}'}{Nu} = D_{IV}'' \quad (86)$$

where the effective Nusselt number is

$$Nu = \alpha d / \bar{\lambda}. \quad (87)$$

(1) Homogeneous Reactors with Turbulent Heat Transfer to the Motor Walls

For the scaling of a homogeneous reactor utilizing a fixed reactant mixture, with thermal and reaction-kinetic similarity, we must satisfy the basic similarity requirements expressed through constancy of (D_I),

$$(D_{III}'), \text{ and } (D_{IV}'').$$

The Prandtl number remains invariant in scaling for a given chemical system. Hence the Nusselt number may be written in the form

$$Nu = \text{constant} (\bar{\rho} d \bar{v} / \bar{\eta})^{m'}$$

where $m' \simeq 0.8$ for turbulent flow. For

$$(\bar{\rho} / \bar{\eta})_H = (\bar{\rho} / \bar{\eta})_M, (\bar{U})_H = (\bar{U})_M, (\vartheta_m - \vartheta_w)_H = (\vartheta_m - \vartheta_w)_M, \\ (\bar{\rho} q' / \bar{\lambda})_H = (\bar{\rho} q' / \bar{\lambda})_M,$$

constancy of (D_{IV}'') now implies the relation

$$\left[\frac{d^2}{(d \bar{v})^{m'}} \right]_H = \left[\frac{d^2}{(d \bar{v})^{m'}} \right]_M. \quad (88)$$

Combining Eq. (88) with the consumption relation

$$d_H^2 \bar{v}_H = n d_M^2 \bar{v}_M$$

shows that

$$d_H = d_M (n)^{m'/(2+m')} \quad (89)$$

and

$$\bar{v}_H = \bar{v}_M (n)^{(2-m')/(2+m')}. \quad (90)$$

From $(D_I)_H = (D_I)_M$ it is then apparent that

$$l_H = l_M (n)^{(2-m')/(2+m')} \quad (91)$$

Equation (91) is also obtained from $(D_{III})_H = (D_{III})_M$ if we introduce the condition

$$(\vartheta_f - \vartheta_o)_H = (\vartheta_f - \vartheta_o)_M.$$

For $m' = 0.8$, Eqs. (89) to (91) become, respectively,

$$d_H = d_M (n)^{0.286}, \quad (89a)$$

$$\bar{v}_H = \bar{v}_M (n)^{0.428}, \quad (90a)$$

$$l_H = l_M (n)^{0.428}. \quad (91a)$$

(2) Heterogeneous Reactors with Constant Pore Size

The similarity parameters which must be maintained constant are

$$D_I' = \frac{l \bar{U}}{\bar{v}}, \quad (92)$$

$$D_{III}' = \frac{q' \bar{U} l}{\bar{v} \bar{c}_p (\vartheta_f - \vartheta_o)}, \quad (93)$$

and

$$D_{IV}' = \frac{\bar{\rho} q' \bar{U} d^2}{\bar{\lambda} (\vartheta_m - \vartheta_w)} \quad (94)$$

where, for turbulent flow through a catalyst reactor, ²

$$\bar{\lambda} = \text{constant } (Re)^{m'} f(d'/d), 0.75 \leq m' \leq 0.97. \quad (95)$$

From the consumption relation we have

$$d_H^2 \bar{v}_H = n d_M^2 \bar{v}_M; \quad (96)$$

from $(D_{IV})_H = (D_{IV})_M$ with $(\bar{U})_H = (\bar{U})_M$, $(\vartheta_m - \vartheta_w)_H = (\vartheta_m - \vartheta_w)_M$, it is apparent that

$$\frac{d_H^2}{\lambda_H} = \frac{d_M^2}{\lambda_M} \quad (97)$$

But, according to Eq. (95),

$$(\bar{\lambda})_H = (\bar{\lambda})_M (\bar{v})_H^{m'} (\bar{v}_M)^{-m'}$$

for constant values of the pore size d' . Hence Eq. (97) becomes

$$\frac{d_H^2}{(\bar{v}_H)^{m'}} = \frac{d_M^2}{(\bar{v}_M)^{m'}} \quad (98)$$

Combining Eqs. (96) and (98) it is seen that

$$d_H = d_M (n)^{m'/(2+2m')} \quad (99)$$

and also

$$\bar{v}_H = \bar{v}_M (n)^{2/(2+2m')} \quad (100)$$

From $(D_I')_H = (D_I')_M$ with \bar{U} constant it is evident that

$$l_H = l_M (\bar{v}_M / \bar{v}_H)$$

or

$$l_H = l_M (n)^{2/(2+2m')} \quad (101)$$

The preceding expressions lead to the following scaling criteria

for maintenance of partial similarity:

(a) $m' = 0.8$:

$$d_H = d_M n^{0.222},$$

$$l_H = l_M n^{0.555},$$

$$\bar{v}_H = \bar{v}_M n^{0.555};$$

(b) $m' = 0.9$:

$$d_H = d_M n^{0.237},$$

$$l_H = l_M n^{0.526},$$

$$\bar{v}_H = \bar{v}_M n^{0.526};$$

(c) $m' = 0$ (laminar flow):

$$d_H = d_M,$$

$$l_H = n l_M,$$

$$\bar{v}_H = n \bar{v}_M.$$

Constancy of D_{II} implies again constancy of $(\mathcal{D}_f - \mathcal{D}_o)$.

(3) Auxiliary Steps in Scaling

It is evident that maintenance of partial similarity, according to the specifications determined by Damköhler and reproduced here, requires auxiliary steps to assure the desired heat transfer in chambers of different sizes. Some considerations of the design changes, as well as a discussion of the influence of radiant heat transfer on motor scaling criteria, may be found in Damköhler's original paper.

The limitations of the application of scaling criteria to chemical reactors have been summarized by Damköhler, who notes particularly the undesirable pressure losses which may be introduced in the rational scaling of heterogeneous reactors. We refer to the original paper for elaboration of these points and conclude the present survey with speculations concerning scaling of bipropellant liquid-fuel rocket engines.

V. SOME SPECULATIONS CONCERNING THE SCALING OF LIQUID-FUEL ROCKET ENGINES

Care must be taken to differentiate between attempts at developing useful scaling criteria without model experiments, on the one hand, and intelligent interpretation and use of even a limited number of experimental data, on the other hand. In the absence of experimental results, we are reduced to the type of considerations presented in Sections III and IV, where we attempt to scale for fixed values of $Y_{i,0}$, \mathcal{G}_0 , \mathcal{D}_0 , and U_i . However,

if experimental data are available on the dependence of U_L on $Y_{i,0}$, ρ_0 , and \mathcal{D}_0 an entirely different approach to the scaling problem is indicated. We proceed by summarizing briefly some conclusions which appear to be justified for the two types of scaling procedures.

A. Scaling Without Model Experiments ($Y_{i,0}$, ρ_0 , \mathcal{D}_0 , and U_L Constant)

It is apparent from the variety of scaling prescriptions obtained in Sections III and IV that any procedures concerning engine scaling for fixed values of $Y_{i,0}$, ρ_0 , \mathcal{D}_0 , and U_L must begin with positive information concerning (a) the importance of heat losses to the motor chamber, and (b) the relative importance of homogeneous gas reactions and of heterogeneous processes such as diffusion flames surrounding liquid droplets.

If heat transfer to the motor walls becomes important, we have again no generally applicable scaling procedures. It remains yet to be shown that Damköhler's notions of partial similarity are useful in practice for liquid-fuel engines. However, it is also clear that in a field as important as that of engine scaling, even an incomplete theory may be a better guide than the artistic notions which are currently used by design engineers.

B. Model Experiments and Scaling

It is clear that a basic development program on engine scaling involves experimental determination of $U_i(p_o, D_o)$ for a fixed chemical system. On the basis of the empirical results it should then be possible to consider rational engine scaling with maintenance of partial or complete similarity. Details concerning this type of investigation had best be carried out in conjunction with the experimental studies.

Analytical considerations form an integral part of the development of scaling procedures from model experiments. In this connection reference should be made to more complete discussions of the chemical processes in reactors than are provided by a similarity analysis.^{9, 10} Correspondingly, a survey of scaling procedures in liquid-fuel rocket engines should begin with attempts at classifying rate-controlling reaction steps,¹ followed by correlation of results concerning past failures and successes on engine scaling.

⁹ R. C. L. Bosworth, *Phil. Mag.* 39, 847 (1948); 40, 314 (1949).

¹⁰ H. M. Hulburt, *Ind. Eng. Chem.* 36, 1012 (1944); 37, 1063 (1945).

As an example of scaling on the basis of model experiments, let it be assumed that in a liquid-propellant engine the rate-controlling reaction involves heterogeneous diffusion flames with $U_i = \text{constant } h^2$, where h is the injector orifice diameter.* The injection orifices are spaced at a distance d' . The total number of injection orifices on a plate of diameter d is proportional to $d^2/(d')^2$ where we assume that $h \ll d'$. If the flow velocity v_0 is chosen to be the linear flow velocity in the orifices, then the total mass flow rate is increased by the factor n if

$$[v_0 h^2 d^2 / (d')^2]_H = n [v_0 h^2 d^2 / (d')^2]_M \quad (102)$$

For constant physico-chemical parameters the following relations obtain:

$$(g_0 v_0 d' / \eta_0)_H = (g_0 v_0 d' / \eta_0)_M, \quad (103)$$

$$(l h^2 / v_0)_H = (l h^2 / v_0)_M, \quad (104)$$

$$(q' h^2 l / v_0 c_{p,0} \vartheta_0)_H = (q' h^2 l / v_0 c_{p,0} \vartheta_0)_M, \quad (105)$$

and, in order to maintain geometric similarity,

$$d_H / d_M = l_H / l_M = d'_H / d'_M = n'. \quad (106)$$

* In general, $\sqrt{U_i}$ is proportional to the droplet diameter h' which, in turn, is given as a function of h , flow velocity v_0 , and pressure drop across the injection orifice. For the implication of this conclusion see pp. 40-41.

Equation (106) assures constant Reynold's number Re based on all chamber dimensions. For constant values of f_o and η_o it follows from Eq. (103) that

$$v_{o,H} d_H' = v_{o,M} d_M'$$

whence

$$v_{o,H} = v_{o,M} / n'. \quad (107)$$

From Eqs. (102), (106), and (107) it is now apparent that

$$h_H^2 = n' n h_M^2. \quad (108)$$

Equations (104), (106) and (107) lead to the result

$$h_H = \frac{1}{n'} h_M. \quad (109)$$

Comparison of Eqs. (108) and (109) shows that

$$(n')^3 = \frac{1}{n}. \quad (110)$$

For $(\mathcal{D}_o)_H = (\mathcal{D}_o)_M$, Eq. (105) is now also satisfied.

The preceding results show that for maintenance of similarity in the special case $U_i = \text{constant } h^2$, eight times the flow rate ($n = 8$) should be achieved by setting $n' = \frac{1}{2}$, i. e., by reducing chamber diameter, chamber length, and orifice spacing by the factor 2, and, at the same time, doubling the linear flow velocity and the size of the orifices.

If $U_i = \text{constant } (h')^2$ then h must be replaced by h' in Eqs. (104) and (105). However, Eqs. (107) and (108) still apply. Equation (109) becomes now

$$h_H' = \frac{1}{n'} h_M'. \quad (111)$$

We must satisfy an auxiliary design requirement corresponding to

$$h' = h'(h, v_0, \Delta p) \quad (112)$$

for a given chemical propellant, where Δp is the pressure drop across the injector orifice. The solution of Eqs. (108), (111), and (112) for suitably chosen values of Δp leads to an exact specification of scaling procedure. Of course, the results can be useful only if the basic relation for U_i applies.

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